

Implications of protective measurements on de Broglie-Bohm trajectories

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Abstract

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Protective measurements which were defined by Aharonov and Vaidman in 1993 [34] played an important role in the discussion about the interpretation of quantum mechanics. In 1999, following an early work by Englert et al. [26] Aharonov et al. [28] wrote an article in which they showed that protective measurements can be used to demonstrate the ‘surrealism’ of Bohmian mechanics. Bohmian mechanics also known as pilot-wave interpretation is certainly the best-known hidden variable interpretation of quantum mechanics. It played a fundamental role in the discovery by Bell of his famous non locality theorem. Therefore, any attacks against pilot-wave interpretation is particularly interesting and instructive to learn something new about the mysterious quantum universe. It is the aim of this chapter to review the debate surrounding protective measurement and pilot-wave (see also [42] and [39]) and to show if it is possible to reconcile the different interpretations of the results given in [28].

AN HISTORICAL REVIEW OF PILOT-WAVE INTERPRETATION

We first remind to the reader some basics about de Broglie-Bohm ‘pilot-wave’ ontology and in particular about its curious history. De Broglie proposed his approach to quantum mechanics in the period 1925-1927, i.e., at the early beginning of modern quantum physics as we know it. De Broglie based his interpretation mainly on relativistic considerations and discovered along this path what is nowadays known as the ‘Klein-Gordon’ equation:

$$\square\Psi(\mathbf{x}, t) = -\frac{m_0^2}{\hbar^2}\Psi(\mathbf{x}, t), \quad (1)$$

What is however puzzling is that the first calculations he did on this subject in 1925 [1] were realized before the discovery by Schrödinger of his famous equation. In some way, we can therefore say that it is quantum wave mechanics which was a development of pilot-wave theory and not the opposite [44].

More precisely, the starting idea of de Broglie [4] was that each single quantum object is actually some highly localized singularity of a specific wave field $\Psi(\mathbf{x}, t)$ which should ultimately be solution of a yet unknown non-linear wave equation. Following Einstein, which had already proposed similar ideas in 1909 [2] for photons (the so called ‘Nadelstrahlung’ concept), de Broglie started a research program baptized ‘double solution’ [5] in which each

quantum is some ‘bunched’ oscillating region of the field propagating as a whole like a particle (i.e. in modern words: a soliton) and inducing a much weaker wave field in its surrounding. This weaker field was supposed to be in ‘harmony of phases’ with the singular field so that both were locked to each other. Following this program the weaker field should obey, far away from the core, a linear equation, e.g., Eq. 1, and subsequently should act as a guiding or pilot wave for the singular part, i.e., determining his complete dynamics. This was of course a very ambitious project and not surprisingly de Broglie never succeeded to complete his theory [12]. Still, during his early quest in 1927 he found a ‘minimalist solution’ which is the foundation of what we call nowadays the de Broglie-Bohm interpretation of quantum mechanics. The theory was introduced at the end of a long article about his double solution program [5] and was subsequently presented during the 5th Solvay congress which took place in Brussels [9] (p. 105-132). In pilot-wave mechanics, the wave is everywhere reduced to its linear contribution, e.g., a solution of Schrödinger equation in the non relativistic regime. The particle behave like a point-like object whose motion is completely determined by the linear wave. De Broglie was able to define the equation of motion of the moving point like particles (for the single and many electron cases) and showed how to solve the dynamic for some specific problems.

Consider for example a single electron described by Schrödinger’s equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x},t) = \frac{-\hbar^2}{2m_0}\Delta\Psi(\mathbf{x},t) + V(\mathbf{x},t). \quad (2)$$

If we know a solution of this equation written in polar form as $\Psi(\mathbf{x},t) = a(\mathbf{x},t)e^{iS(\mathbf{x},t)/\hbar}$ we can define a density of probability $\rho(\mathbf{x},t) = \Psi(\mathbf{x},t)\Psi(\mathbf{x},t)^*$, i.e., $\rho(\mathbf{x},t) = a(\mathbf{x},t)^2$ and a probability current $\mathbf{J}(\mathbf{x},t)$ such as

$$\mathbf{J}(\mathbf{x},t) = \hbar\frac{\Psi(\mathbf{x},t)^*\nabla\Psi(\mathbf{x},t) - \Psi(\mathbf{x},t)\nabla\Psi(\mathbf{x},t)^*}{2im_0} = a(\mathbf{x},t)^2\frac{\nabla S(\mathbf{x},t)}{m_0}. \quad (3)$$

Using these equations de Broglie defined the velocity of the particle as

$$\mathbf{v}(t) = \frac{d}{dt}\mathbf{x}(t) = \frac{\mathbf{J}(\mathbf{x},t)}{\rho(\mathbf{x},t)} = \frac{\nabla S(\mathbf{x},t)}{m_0}, \quad (4)$$

showing that in analogy with classical dynamics $S(\mathbf{x}(t),t)$ plays the role of an action (see also Madelung [6]). This analogy is even enforced when we insert a and S in Eq. 2 to obtain

$$-\frac{\partial}{\partial t}S(\mathbf{x}(t),t) = \frac{(\nabla S(\mathbf{x},t))^2}{2m_0} + V(\mathbf{x}(t),t) - \frac{\hbar^2\Delta a(\mathbf{x}(t),t)}{2m_0a(\mathbf{x}(t),t)}. \quad (5)$$

We recognize the well-known Hamilton-Jacobi equation which in classical dynamics determines the motion of the particle in an external potential V . However, there is here an additional term $Q(\mathbf{x}, t) = -\hbar^2 \Delta a(\mathbf{x}, t) / (2m_0 a(\mathbf{x}, t))$ called the quantum potential by de Broglie. This potential is determined by the wave amplitude in agreement with the pilot-wave idea. Importantly Q is unchanged if the wave function is multiplied by a constant so that actually it is the form of the wave more than its amplitude which has a signification in this theory. Also, for a many body system the potential $Q(x_1, x_2, \dots, x_N, t)$ depends in general in a nonlocal way of the N particle coordinates. This can lead to some specific features such as non local entanglement discussed in the context of the EPR paradox [7] or the Bell inequality [8]. In particular, the fact that pilot-wave theory agrees with Bell's theorem implies some kind of mysterious action at a distance between the particles. We point out that de Broglie contrary to Bohm was very reluctant to introduce non locality in his ontological theory and that he expected to remove this feature with his double solution program.

Importantly, the Hamilton-Jacobi analogy suggests that pilot-wave theory can equivalently be written in Newton's form. The second law for de Broglie's dynamics is indeed easily written as $m_0 \frac{d^2}{dt^2} \mathbf{x}(t) = -\nabla[V(\mathbf{x}(t), t) + Q(\mathbf{x}(t), t)]$ in full analogy with classical dynamics for a point-like particle. However, while this dynamical law contains a second order time derivative it is important to observe that for practical purposes if $\Psi(\mathbf{x}, t)$ is known then the first order Eq. 4 is sufficient to completely describe the trajectories. This is indeed done through integration of the flow equations:

$$\frac{dx}{\frac{\partial S(\mathbf{x}, t)}{\partial x}} = \frac{dy}{\frac{\partial S(\mathbf{x}, t)}{\partial y}} = \frac{dz}{\frac{\partial S(\mathbf{x}, t)}{\partial z}} = \frac{dt}{m_0}. \quad (6)$$

for a given initial condition $\mathbf{x}(t_0) = \mathbf{x}_0$. This point is important because John Bell [8] used pilot-wave theory mainly through the definition given by Eq. 4 while other authors like Vigier [19] and Bohm [20] insisted on the need to use the quantum potential for a complete physical description of the particle motion. This seems to indicate that the theory lacks a univocal axiomatic for his foundation.

At Solvay conference W. Pauli was probably the most reactive concerning criticisms but even potential followers like Einstein or the more 'classical' Lorentz were not showing a too strong enthusiasm for de Broglie pilot-wave approach. Remarkably, due to internal mathematical difficulties of his 'double solution' program de Broglie only presented his pilot-wave

version in Brussels. This was certainly an honest choice but physically far less profound and less impressive for this demanding audience. In particular, one of Pauli's objection concerned the arbitrariness of the dynamics law obtained by de Broglie. Indeed, Pauli observed [9] (see p. 134-135) that the dynamics proposed by de Broglie has no precise foundation since the conservation current is not univocal, i.e. one can add a divergence free vector to \mathbf{J} without changing the conservation law, and Schrödinger asked further why we should not use instead of Eq. 4 a different definition [9] (p. 135), e.g., the energy-momentum tensor $T^{\mu\nu}(\mathbf{x}, t)$ in order to define a trajectory. This was indeed proposed by de Broglie himself for photons [10] (see however [13] for a modern perspective concerning this problem and the difficulties about a covariant generalization of pilot-wave).

We also mention a related critical comment concerning foundation made in 1952 by Pauli [11] and in 1955 by Heisenberg [15]. Both physicists indeed complained by observing that for de Broglie and Bohm the particle position plays a fundamental role that breaks the accepted a symmetry between position and momentum (symmetry which is at the heart of quantum formalism). This was unacceptable for Heisenberg, and Pauli, for whom position q and momentum p should be introduced at an equal footing.

Of course, all these observations by Pauli, Schrödinger and Heisenberg are not decisive remarks against pilot-wave interpretation since the plausibility or un-plausibility of the dynamics doesn't constitute by itself a proof or disproof of the theory: only experiments should have the last word. Nevertheless, altogether these problems let a strong feeling of discomfort to the audience of the Solvay conference and to the first generations of quantum theorists. This discomfort never really disappeared until nowadays.

THE MEASUREMENT THEORY AND THE ADIABATIC THEOREM

Einstein's reaction

Beyond these interesting problems about axiomatics and foundations the most critical part of the theory concerns of course his agreement with experimental facts and the realism of the predictions given by the pilot-wave approach. Indeed, if Schrödinger's equation completely determines the particle motion through Eq. 4 then we expect that both the usual 'Copenhagen' approach and the one of de Broglie should be experimentally equivalent. This

was indeed later confirmed after the more detailed studies of measurement processes by David Bohm in 1952 [21]. Still, in 1927 de Broglie [5] already showed that the four-vector current J^μ , which naturally arises from wave equation 1 and formally leads to the conservation law $\partial_\mu J^\mu = 0$ through the Noether theorem, can be used to justify the statistical interpretation of quantum mechanics, i.e., the so called ‘Born’s probability rule’. Indeed, if for simplicity we limit ourself to the non relativistic regime then, the evolution equation Eq. 4 and the current conservation rule $\partial_t \rho + \nabla \cdot \mathbf{J} = 0$, imply the following: if at a given time the probability distribution of particle in space is given by a^2 then this will also be true at any time. If we write $a^2(\mathbf{x}(t_0), t_0)$ the density of probability at $\mathbf{x}_0 = \mathbf{x}(t_0)$ and time t_0 we can obtain by direct integration the density of probability at time t for the point $\mathbf{x}(t)$ located along the de Broglie trajectory (see Eqs. 4,6). We get

$$a^2(\mathbf{x}(t), t) = a^2(\mathbf{x}(t_0), t_0) \cdot e^{-\int_{t_0}^t dt' \frac{\Delta' S(\mathbf{x}(t'), t')}{m_0}}, \quad (7)$$

where $\Delta' = \partial^2 / \partial \mathbf{x}(t')^2$. This is the same reasoning which is used in classical statistical mechanics, e.g., Liouville, Gibbs, to justify probability laws. In particular, if at given time the wave functions of particles can be approximated by uncorrelated plane waves then and ‘apriori’ symmetry implies the homogeneity of probability distribution in space (this can be seen as an initial chaotic condition à la Bolstzmann, i.e., a ‘Stosszahlansatz’). The subsequent interaction processes between the different particles will certainly create correlations between them but then the deterministic evolution (e.g., Eq. 4 and its generalization for the many-body problem) will maintain the probability interpretation for any other time t as we already said. This idea was further developed by Bohm [18], Vigier [19] and Nelson [22] in the 50-60’s and more recently by Valentini [23];[24], Dürr, Goldstein and Zanghi [25] with different strategies.

This is certainly impressive, or at least promising, but the theory possesses some other ‘repellant’ features which were studied in the recent years and are the subject of the present chapter. One of them already mentioned by Ehrenfest in [9] p. 136 concerns the fact that in the ground state of an Hydrogen atom (i.e. a s state) the wave function is (up to the $e^{-iEt/\hbar}$ contribution) real. It implies $\mathbf{v} = \nabla S / m_0 = 0$ i.e, the fact that the electron is at rest in the s-atom. From the point of view of the de Broglie theory there is nevertheless no contradiction since the constant energy E is given by $E = -\partial_t S = V(\mathbf{x}) + Q(\mathbf{x})$ and the variation of Q with \mathbf{x} exactly compensates the variation of V . The force $\mathbf{F} = -\nabla[V + Q]$

therefore vanishes and the electron is not accelerated. Still, this feature looked not realistic and played again against de Broglie. Not surprisingly, after this period 1927-1928 de Broglie abandoned his theory and went back to it only after 1952 and the rediscovery by Bohm of pilot-waves. We point out that the ‘ $\mathbf{v} = \nabla S/m_0 = 0$ ’ objection played also a role in the ‘cold’ reception of this theory by Pauli [11] and Einstein and this even after 1952 (see also Rosen [27] who re-discovered, after de Broglie but before Bohm, the pilot-wave concept and repudiated it for the same reasons as Einstein). In a paper written for Max Born retirement from the University of Edinburgh [16] Einstein discussed the example of a particle in a infinite 1D potential well which admits wave functions

$$\Psi(x, t) = \sqrt{\frac{2}{L}} \sin(n\pi x/L) e^{-iE_n t/\hbar} \quad (8)$$

associated with the energy $E_n = (\hbar n\pi/L)^2/(2m_0)$ for $n = 0, 1, 2$, etc. Clearly, here again the velocity of the particle cancels. For Einstein this seemed to contradict the fact that for large n an ontological theory like pilot-wave should ‘intuitively’ recover classical mechanics. However, in classical mechanics we have $Q = 0$ and $E = p^2/(2m_0)$ with $p = m_0 v$. This apparently fits well with Schrödinger equation if we write $p = \hbar n\pi/L$. Unfortunately, pilot-wave of de Broglie and Bohm implies $p = m_0 v = \nabla S = 0$ and $Q = (\hbar n\pi/L)^2/(2m_0) = E$. Most remarkably, this occurs independently of how large the quantum number n is and is therefore in complete contradiction with what we intuitively expect in the classical regime. Commenting further on Bohm’s attempt to reintroduce pilot-wave theory Einstein once wrote to Born ‘That way seems too cheap to me’. Still, we point out that neither de Broglie nor Bohm agreed with Einstein’s conclusion. For example, in his book written with B. Hiley [45] Bohm replied that, independently of the details of pilot-wave theory, any model attempting to preserve the particle localization in the infinite potential well would ultimately contradicts classical physics. This should be the case since at fixed energy there are necessarily some nodes where the wave function cancels and are therefore prohibited to the particle localization, i. e., corresponding to regions where the probability is zero. In the 1D case the potential well is thus obviously separated into small spatial cells of size $\lambda/2 = L/n$ where the particle is confined and cannot escape because it cannot cross or even reach the nodes. Therefore, in this context the expectation of Einstein appears illusory. Still, the example of Einstein or the one of the s atom constitute perfect illustrations of the ‘surrealistic nature of de Broglie-Bohm trajectory’. This qualifier was given by Englert *et*

al. after a very detailed paper [26] which discussed pilot-wave interpretation in the context of measurement theory.

Von Neumann's strong measurements

The most important contribution of David Bohm to pilot-wave theory concerns his interpretation of quantum measurements. In 1952 in a series of two well known papers [20];[21] he discussed the canonical von Neumann projective measurements in the context of pilot-wave theory. He showed that there is nothing of contradictory or impossible in attributing at the same time a position and a momentum to a particle as soon as we accept that the so-called momentum measured is not in general its actual momentum. This should be already clear from the definition $\mathbf{p}(t) = m_0 \mathbf{v}(t) = \nabla S$ which holds at any location \mathbf{x} visited by the particle. The plane-wave eigenstates $|\mathbf{p}\rangle$ of the operator $\hat{\mathbf{p}} = -i\hbar \nabla$ are completely delocalized and according to Heisenberg principle this prohibits a clean localization of the particle. The agreement between the definition of de Broglie-Bohm on the one side and of Heisenberg on the other side is reestablished if we realize that in order to measure the momentum associated with the operator $\hat{\mathbf{p}}$ one must disturb the initial wave function and separate the different plane wave contributions.

Consider once again the example of the infinite potential well. Bohm observes that the wave function given by Eq. 8 can be formally expanded into plane waves and the Fourier amplitudes $\tilde{\psi}(p)$ correspond to two well localized wave packets peaked near the momentum values $p = \pm \hbar n/L$ (in the classical limit $n \rightarrow +\infty$ we have $|\tilde{\psi}(p)|^2 \simeq [\delta(p - \hbar n/L) + \delta(p + \hbar n/L)]/2$). Bohm then supposed that the walls or the well confining the particle instantaneously disappears without disturbing in any appreciable fashion the wave function. The two wave packets subsequently propagate freely and ultimately separate from each other. This makes the packets spatially distinguishable and allows for a measurement of the particle momentum $p \simeq \pm \hbar n/L$.

This example is of course a ‘gedanken’ experiment and subsequent studies made by Bohm and followers focussed on the procedure of entanglement between a pointer or meter and the analyzed quantum system.

This was first done in the context of von Neumann measurement whose method was well discussed by Bohm himself in his ‘orthodox’ 1951 text book, e.g., for the Stern-Gerlach

experiment analysis [14]. The main idea can be easily illustrated by considering the total Hamiltonian

$$\hat{H}(t) = \hat{H}_S + \hat{H}_M - \hbar g(t) \epsilon \hat{A}_S \hat{X}_M, \quad (9)$$

describing the interaction between a system S and a meter M. The operator \hat{A}_S acts only on S and corresponds to the variable we wish to measure. \hat{X}_M is the operator describing the meter. It represents here its position (e.g. the atom center of mass in the Stern-Gerlach experiment). The coupling is also characterized by a constant ϵ introduced for the sake of the equation homogeneity and a time dependent function $g(t)$ characterizing the fast evolution of the measurement protocol. Here, we impose to simplify $g(t) = \delta(t)$, i.e., an instantaneous measurement. Before the interaction occurs at $t = 0$ we start, i.e., for $t < 0$, with two decoupled and unentangled subsystems S and M described by the quantum state $|\Psi_{in}(t)\rangle = |S(t)\rangle \otimes |M(t)\rangle$. After the interaction occurred, i.e. for $t > 0$, we obtain the final state $|\Psi_f(t)\rangle = \hat{U}_S(t, t=0) \hat{U}_M(t, t=0) |\Psi_f(0)\rangle$ where $\hat{U}_S(t, t=0)$ and $\hat{U}_M(t, t=0)$ are the evolution operators of the freely moving subsystems S and M acting on

$$\begin{aligned} |\Psi_f(0)\rangle &= e^{i\epsilon \hat{A}_S(0) \hat{X}_M(0)} |S(0)\rangle \otimes |M(0)\rangle \\ &= \sum_a \int dp S(a) M(p) e^{i\epsilon a \hat{X}_M} |a\rangle \otimes |p\rangle \end{aligned} \quad (10)$$

or equivalently on

$$\begin{aligned} |\Psi_f(0)\rangle &= \sum_a \int dp S(a) M(p) |a\rangle \otimes |p + \hbar \epsilon a\rangle \\ &= \sum_a \int dp S(a) M(p - \hbar \epsilon a) |a\rangle \otimes |p\rangle. \end{aligned} \quad (11)$$

Here we used the expansion of the initial wave packets in the vector basis $|a\rangle$ and $|p\rangle$ respectively and applied well-known properties of the translation operators $\hat{T}(\hbar a) = e^{i\epsilon a \hat{X}_M}$. In particular if we take $M(p) = e^{-\Delta p^2}$ we obtain after the measurement a series of shifted gaussians $M'(p) = e^{-\Delta(p - \hbar \epsilon a)^2}$ entangled with each state $|a\rangle$. If the shift of each gaussian is larger than their typical width $\delta p = 1/(2\Delta)$ (and if we can neglect the free space spreading of the pointer wave packets) it will be possible to correlate the distribution $|S(a)|^2$ of S with the distribution of gaussian centers in the momentum space of M. This is the basis of the von Neumann measurement protocol which was translated into the ontological language of pilot-wave theory by Bohm in 1952. For Bohm indeed, the entanglement directly affected the

particles trajectories of the two subsystems S and M but if we observe the meter at a location near $p \simeq \hbar \epsilon a$ it does not however implies that S is actually in the state a . This apparently paradoxical result comes from the fact that in pilot-wave theory the position of particles plays a more fundamental role than in the usual interpretation. Therefore, one should be authorized to speak about measurement only if we can correlate the studied variables a with the actual position of the system S. Interestingly, both interpretations by von Neumann and Bohm of the previous protocol will however eventually agree if the different wave packets of the subsystem S: $\psi_a(x_S)$ in the base a are not spatially overlapping. In a more general way, if the entanglement between the system S and meter M produces after the interaction a sum of entangled states $\sum_i c_i \psi_i(x_S) \phi_i(x_M)$, where the different wave functions for both particles are non overlapping, we will then unambiguously be able to correlate the positions of S and M with the states labeled by i . For most experiments this is however not the case and the so-called quantum measurement cannot be considered as such in the context of pilot-wave theory. It is therefore amazing to observe that the famous dictum of Wheeler ‘No elementary phenomenon is a phenomenon until it is an observed phenomenon’ which was given in the context of Bohr’s interpretation finds also his plain significance in the interpretation of de Broglie and Bohm. Paraphrasing Wheeler, we could then state that ‘No measurement is a measurement until it is a position measurement’. It is also worth mentioning that in the same texts quoted previously, both Pauli [11] and Heisenberg [15] criticized this strange feature of pilot-wave approach. Heisenberg, in particular, pertinently commented that in the deterministic approach of Bohm momentum and position are in general hidden and correspond therefore to metaphysical superstructures without any physical implication.

Protective measurements

The previous discussion done in the context of orthodox von Neumann strong projective measurements was extended in 1999 to the so-called weak protective measurement domain by Aharonov, Englert, and Scully in a fascinating paper [28]. The authors showed that in the considered regime the interpretation by pilot-wave of the results implied some even more drastic surrealism as in the strong coupling regime. To understand their motivation it is important to go back once again to the origin of pilot-wave mechanics and to observe that if the wave guides the particle during its motion then in some situations empty waves without

particle should exist. For example, in the double slit experiment the particle travels through one hole but something should go through the second hole in order to disturb the motion on the other side and induce interference. Of course, one can always involve the quantum potential as an explanation but then one should explain why this potential exists and the problem is therefore not removed. Many authors thinking about this problem claimed that the guiding wave should carry some energy and the particle should get less and less energy while crossing an interferometer with more and more gates and doors [12]. But obviously, this is not what is predicted neither by quantum mechanics nor pilot-wave theory. Another point, was that if empty wave reacts on the particle during the double-slit interference experiment why should it not also acts on some other systems [17]. Could we detect an empty wave? While working on this problem it was realized by L. Hardy [29];[30];[31] that empty waves can sometimes have a physical effect on a second entangled (measuring) system (his idea was actually an adaptation of Elitzur and Vaidman ‘interaction free-measurement’ protocol [32]) and he found during his research a very fascinating Bell’s theorem without inequality involving strange non local features and questioning the possibility to build up a Lorentz invariant hidden variable model. The result of Hardy is intriguing and also disappointing since, again, it is an indirect effect on hidden variables which is observed. The empty wave affects the dynamics of the second system but one must watch correlations between events to see it (otherwise one could send faster-than-light signals with this nonlocal protocol). For those already not convinced by pilot-wave approach this definitely could not help. In a different but related context J. Bell in 1980 [8] (p. 111-116) studied the exotic behavior of Bohmian particles diffracted by a screen and interacting with a complex detecting ‘which-path’ device. It was shown that the path followed by the particle is sometime completely surrealistic and can even reach the wrong detector (this is connected to the fact that Bohmian trajectory cannot cross in the configuration space). However, this cannot affect the interpretation since this is again hidden and impossible to test experimentally. In a subsequent paper by Englert et al. [26], already mentioned (see also [40] and [46]) it was shown that the problem is deeper than Bell thought at first, and that this surrealism exists even with simple particles interacting with Stern and Gerlach devices [41]. Therefore, to quote the authors :‘the reality attributed to Bohm trajectories is not physical it is meta-physical’ [26]. Lev Vaidman [33] wrote once a very pedagogical paper provocatively untitled: ‘The reality in Bohmian quantum mechanics or can you kill with an empty wave bullet’.

In his paper, Vaidman explained with very symptomatic and illustrative examples (such as slow bubble traces developing after the passage of the particle even when the particle is not here but elsewhere) that if one is living in pilot-wave world then entanglement with meters and environment will break all your convictions about causality and localization (i.e. in agreement with Hardy's conclusions).

His paper reviewing the argument presented in [28] showed also that if the empty waves are involved in all these processes then one can actually measure an empty wave function without the particle being. This relies on protective measurements of position with allow a measure of the wave function density $|\psi(x)|^2$ of the particle at x even if pilot-wave trajectory never crosses the interaction region centered on x . The concept of protective measurement is a beautiful idea which was introduced by Y. Aharonov and L. Vaidman [34];[35] (see also [36] and [33]). The principle relies on the possibility to couple adiabatically the measuring device M with the subsystem S in such a way to induce no significant change in the $|S(0)\rangle$ initial state while disturbing the meter state $|M(0)\rangle$ in an observable fashion. In such an approach, the system S is therefore protected and it is easily shown that one can use this kind of protocol to record an information on some local observable such as $|\psi(x)|^2$ or $\mathbf{J}(x)$. The specific example considered in [28] is based, once more, on the infinite potential well but now with a very local interaction with a meter at one point (i.e. $0 < x = x_0 < L$) of the cavity. The total Hamiltonian is

$$\hat{H}(t) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{-\hbar^2}{2M} \frac{\partial^2}{\partial X^2} - \hbar \epsilon g(t) \delta(x - x_0) X \quad (12)$$

where x is the coordinate of the particle of mass m in the box while X is the coordinate of the meter with mass $M \gg m$. The coupling is monitored by the external parameter $g(t)$ such as $\int_{-\infty}^{+\infty} dt g(t) = 1$. If $g(t)$ changes very fast one goes back to the von Neumann regime but here $g(t)$ changes very slowly, i.e., adiabatically, and it vanishes outside the interval $[-T/2, +T/2]$ where it has the typical value $g(t) \simeq 1/T$. The most characteristic feature of this interaction is of course the presence of the Dirac function which implies a short-range coupling existing only in the vicinity of $x = x_0$. In order to solve the dynamical equation we apply here the adiabatic approximation method [14] and we first search for eigenstates of the equation $\hat{H}(t)\Psi(x, X, t) = E(t)\Psi(x, X, t)$. Inserting the 'ansatz' $\Psi(x, X, t) = \phi_s(x, X, t)e^{iPX/\hbar}/\sqrt{(2\pi\hbar)}$ we get the new equation:

$$[E(t) - \frac{P^2}{2M}]\phi_s(x, X, t) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \phi_s(x, X, t) - \hbar \epsilon g(t) \delta(x - x_0) X \phi_s(x, X, t). \quad (13)$$

This is actually a 1D Green function problem with t and X as parameters. and Aharonov et al. solved it analytically [28]. Still, since we suppose the coupling to be weak we can alternatively use (as they did as well) the first-order perturbation approximation which leads to: $\phi_s(x, X, t) \simeq \phi_n(x) = \sqrt{\frac{2}{L}} \sin(n\pi x/L)$ and $E_{n,P}(X, t) - \frac{P^2}{2M} = (\hbar n\pi/L)^2/(2m) + \delta E$ with

$$\delta E = -\hbar \epsilon g(t) X \langle n | \delta(\hat{x} - x_0) | n \rangle = -\hbar \epsilon g(t) X |\phi_n(x_0)|^2. \quad (14)$$

We point out that there is actually a small slope discontinuity at x_0 since for the 1D Green function we must have:

$$\begin{aligned} d\phi_s(x, X, t)/dx|_{x_0+\delta} - d\phi_s(x, X, t)/dx|_{x_0-\delta} \\ = -2m\epsilon g(t) X \phi_s(x_0, X, t)/\hbar \end{aligned} \quad (15)$$

with $\delta \rightarrow 0^+$. In the weak coupling regime we can neglect this effect and therefore the cavity mode can fairly be considered as ‘protected’. The next step is to expand the full system wave function by solving the Schrödinger equation $i\hbar d\Psi(t)/dt = H(t)\Psi(t)$ and using these eigenmodes labeled by the index n of the cavity mode (here we will limit our analysis to $n = 1$) and P the ‘orthodox’ momentum of the pointer. We have :

$$\Psi(x, X, t) = \sum_{n,P} b_{n,P}(t, X) \Psi_{n,P}(x, X, t). \quad (16)$$

In the adiabatic approximation we write the amplitude coefficients as

$$b_{n,P}(t, X) = c_{n,P}(t, X) e^{-i \int_{-\infty}^t dt' E_{n,P}(X, t')/\hbar} \quad (17)$$

and we get here:

$$\begin{aligned} \Psi(x, X, t) \simeq \phi_s(x, X, t) \int \frac{dP}{\sqrt{(2\pi\hbar)}} M(P) e^{iPX/\hbar} e^{-i \frac{P^2}{2M\hbar} t} \\ \cdot e^{i \int_{-\infty}^t dt' [\epsilon g(t') X |\phi_n(x_0)|^2 - \beta_n(t')]} e^{-i E_n t/\hbar} \end{aligned} \quad (18)$$

with $i\beta_n(t) = \langle n | \frac{d}{dt} | n \rangle \simeq 0$. In doing this calculation we supposed the initial state begin unentangled, i.e., like for the von Neumann procedure. This initial state corresponds to the product of a undisturbed cavity mode $n = 1$ (i.e. $\phi_s(x, X, t) \simeq \sqrt{\frac{2}{L}} \sin(\pi x/L)$) by a localized wave packet with gaussian Fourier coefficient $M(P) \propto e^{-\Delta p^2}$. The coupling is

supposed to be slow and weak so that the energy given by the interaction is not large enough to induce transition between different eigenmodes. For $t \rightarrow +\infty$ we thus get

$$\Psi(x, X, t) \simeq \phi_s(x, X, t) e^{-iE_n t/\hbar} \cdot \Psi_M(X, t) \cdot e^{i\epsilon X |\phi_n(x_0)|^2} \quad (19)$$

which shows that the main result of the interaction is to induce a phase kick to the pointer wave packet $\Psi_M(X, t)$. If we neglect the free space spreading of the pointer wave packet this phase shift will impose a translation $\Delta P \simeq +\hbar\epsilon |\phi_n(x_0)|^2$ in the Fourier space such as $M'(P) = M(P - \Delta P)$. This results in a protective measurement where the local adiabatic coupling keeps the confined mode $\phi_s(x, X, t)$ undisturbed.

Now comes the paradox: since the cavity mode is protected and since it corresponds to a de Broglie vanishing velocity of the particle S (i.e., $dx(t)/dt = \partial_x S(\mathbf{x}, t)/m = 0$) we deduce that the pointer M is disturbed by the local interaction centered at $x = x_0$ even though S never approaches this position. How could that be? For Aharonov et al. one can hardly avoid the conclusion that Bohmian trajectories are just a mathematical construct. The same conclusion was actually given (although in a less technical way) in a previous paper [37] where the authors concluded that Bohmian trajectory contradicts the natural statement: ‘an empty wave should not yield observable effects on other particles’. Indeed, the measuring device recording $|\psi_n(x_0)|^2$ in the ‘empty’ region surrounding x_0 yields non-null outcomes (identical conclusions were discussed in [38]). In his review paper [33] Vaidman however considered the problem from a wider perspective and commented that for him in the framework of Bohmian mechanics there is no fundamental problem since ‘these experiments are *not* good verification measurements’ so that Bohmian proponents have ‘a good defense’. Nevertheless, this looks mysterious or magical since one would like to find where does the force acting on the pointer come from? Furthermore, even if one is not accepting the ontology proposed by de Broglie’s pilot-wave it was at least possible until now to accept its self-consistency. Does protective measurement changes the rules? Indeed, magical forces have no place in physics. In order to remove some of these ambiguities and magical features I developed in a paper published in 2005 [42] a dynamical analysis of the protective measurement discussed in [28] seen from the point of view of pilot-wave theory. I will now summarize my reasoning using the calculations given before. First, we observe that the quantum potential for the system

given by Eq. 19 is :

$$\begin{aligned} Q(x, X, t) &= \frac{-\hbar^2}{2m} \frac{\frac{\partial^2}{\partial x^2} |\Psi(x, X, t)|}{|\Psi(x, X, t)|} + \frac{-\hbar^2}{2M} \frac{\frac{\partial^2}{\partial X^2} |\Psi(x, X, t)|}{|\Psi(x, X, t)|} \\ &\simeq \frac{-\hbar^2}{2m} \frac{\frac{\partial^2}{\partial x^2} |\phi_s(x, X, t)|}{|\phi_s(x, X, t)|} + \frac{-\hbar^2}{2M} \frac{\frac{\partial^2}{\partial X^2} |\Psi_M(X, t)|}{|\Psi_M(X, t)|}. \end{aligned} \quad (20)$$

Here, we fairly neglected the small contributions of terms containing the X derivatives of $|\phi_s(x, X, t)|$. Now using Eqs. 13, 14 and the fact that $\phi_s(x, X, t) \simeq \phi_1(x)$ is real we immediately get

$$\begin{aligned} Q(x, X, t) &\simeq \frac{(\hbar\pi/L)^2}{2m} - \hbar\epsilon g(t)X|\phi_1(x_0)|^2 \\ &+ \hbar\epsilon g(t)\delta(x - x_0)X + \frac{-\hbar^2}{2M} \frac{\frac{\partial^2}{\partial X^2} |\Psi_M(X, t)|}{|\Psi_M(X, t)|}. \end{aligned} \quad (21)$$

Now, the potential acting in the Hamilton-Jacobi equation is $U = V + Q$ where $V = -\hbar\epsilon g(t)\delta(x - x_0)X$ is the ‘classical’ local interaction potential associated with the protective measurement protocol. Here, this leads therefore to

$$U(x, X, t) \simeq \frac{(\hbar\pi/L)^2}{2m} - \hbar\epsilon g(t)X|\phi_1(x_0)|^2 + \frac{-\hbar^2}{2M} \frac{\frac{\partial^2}{\partial X^2} |\Psi_M(X, t)|}{|\Psi_M(X, t)|}. \quad (22)$$

Remarkably, the local potential has been removed from the total Hamiltonian because the singular term in V exactly compensates the one in Q . This implies that from the framework of pilot-wave theory the interaction is highly quantum-like, i.e., it has not classical analog. This is even more clear in the Newton picture. Newton’s law reads indeed $md^2x(t)/dt^2 = F_x$ and $md^2X(t)/dt^2 = F_X$ and with the definition for U this implies for the evolution of S :

$$F_x = -\frac{\partial}{\partial x}U(x, X, t) \simeq 0 \quad (23)$$

i.e. the force applied on the Bohmian particle vanishes. This situation is exactly similar to the one obtained in the Einstein example or in a state atom discussed by de Broglie, Pauli and Einstein. In each cases the quantum potential is constant over the region of interest so that the particle can indeed stay in static equilibrium in full agreement with the de Broglie guidance condition $mdx(t)/dt = \partial_x S(x, X, t) = 0$. The big difference is that in the protective measurement there is actually a local force $-\frac{\partial}{\partial x}V$ but its effect is compensated by an additional quantum term in $-\frac{\partial}{\partial x}Q$. Remarkably, the situation is completely different for the meter M since we get:

$$F_X \simeq -\frac{\partial}{\partial X}U(x, X, t) \simeq +\hbar\epsilon g(t)|\phi_1(x_0)|^2 \quad (24)$$

in agreement with the momentum kick $\Delta P = \int dt' F_X(t') = +\hbar\epsilon|\phi_1(x_0)|^2$ introduced previously. Therefore, the pointer deviation is completely justified from the point of view of de Broglie and Bohm approach. However, here the force applied on M is of quantum origin and not the local and classical term $-\frac{\partial}{\partial x}V$.

A SHORT CONCLUSION

Finally, what can we deduce from this story? We reviewed pilot-wave theory and showed that the surrealism objection is very old and goes back to the origin of the theory. Einstein did not like this theory in part because the trajectories predicted in general don't follow our classical intuitions about dynamics. Latter, this surrealism was criticized because very often even causality is affected by pilot-wave. This of course included non locality as studied by Bell but also modifications of our intuitions about what should a trajectory in an interferometer be. The work by Aharonov et al. on protective measurements follows this strategy, and indeed, it confirms that pilot-wave is not classical. Still, this theory is the only known quantum ontology (Lev Vaidman will certainly not agree here) which is completely self consistent at the mathematical level and at the same time explains every experimental fact (too many words could be said here about the Everett's interpretation [43] and its problems associated with probabilities and this will be therefore omitted). Of course, it is probably only a temporary expedient and pilot-wave theory has no convincing or univocal relativistic generalization, but to quote Bell 'Should it not be taught, not as the only way, but as an antidote to the prevailing complacency?' [8] (see p. 160).

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